

Activity 2 Mark scheme

Question Number	Scheme	Marks
3. (i)	$(x-4)^2 \geq 2x-9 \Rightarrow x^2 - 10x + 25 \dots 0$ $\Rightarrow (x-5)^2 \dots 0$ Explains that "square numbers are greater than or equal to zero" hence (as $x \in \mathbb{R}$), $\Rightarrow (x-4)^2 \geq 2x-9$ *	M1 A1 A1* (3)
(ii)	Shows that it is not true for a value of n Eg. When $n = 3$, $2^n + 1 = 8 + 1 = 9 \times$ Not prime	B1 (1) (4 marks)
Notes		
(i)	A proof starting with the given statement	
M1	Attempts to expand $(x-4)^2$ and work from form $(x-4)^2 \dots 2x-9$ to form a 3TQ on one side of an equation or an inequality	
A1	Achieves both $x^2 - 10x + 25$ and $(x-5)^2$. Allow $(x-5)^2$ written as $(x-5)(x-5)$	
A1*	For a correct proof. Eg "square numbers are greater than or equal to zero", hence (as $x \in \mathbb{R}$), $(x-5)^2 \geq 0$ $\Rightarrow (x-4)^2 \geq 2x-9$ This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers and (3) a reference back to the original statement Answers via $b^2 - 4ac$ are unlikely to be correct. Whilst it is true that there is only one root and therefore it touches the x-axis, it does not show that it is always positive. The explanation could involve a sketch of $y = (x-5)^2$ but it must be accurate with a minimum on the +ve x axis with some statement alluding to why this shows $(x-5)^2 \geq 0$	
Approaches via odd and even numbers will usually not score anything. They would need to proceed using the main scheme via $(2m-4)^2 \geq 4m-9$ and $(2m-1-4)^2 \geq 2(2m-1)-9$		
Alt to (i) via contradiction		
Proof by contradiction is acceptable and marks in a similar way		
M1	For setting up the contradiction 'Assume that there is an x such that $(x-4)^2 < 2x-9 \Rightarrow x^2 - 10x + 25 \dots 0$	
A1	$\Rightarrow (x-5)^2 \dots 0$ or $(x-5)(x-5) \dots 0$	
A1*	This is not true as square numbers are always greater than or equal to 0, hence $(x-4)^2 \geq 2x-9$	
Alt to part (i) States $(x-5)^2 \geq 0$ $\Rightarrow x^2 - 10x + 25 \geq 0$ $\Rightarrow x^2 - 8x - 16 \geq 2x-9$ $\Rightarrow (x-4)^2 \geq 2x-9$		

Question Number	Scheme	Marks
M1	States $(x-5)^2 \geq 0$ and attempts to expand. There is no explanation required here	
A1	Rearranges to reach $x^2 - 8x - 16 \geq 2x - 9$	
A1*	Reaches the given answer $(x-4)^2 \geq 2x - 9$ with no errors	
.....		
(ii)		
B1	Shows that it is not true for a value of n This requires a calculation (and value found) with a minimal statement that it is not true Eg. ' $2^6 + 1 = 65$ which is not prime' or ' $2^5 + 1 = 33 \times$ ' Condone sloppily expressed proofs. Eg. ' $2^7 + 1 = \frac{129}{3} = 43$ which is not prime' Condone implied proofs where candidates write $2^5 + 1 = 33$ which has a factor of 11 If there are lots of calculations mark positively. Only one value is required to be found (with the relevant statement) to score the B1 The calculation cannot be incorrect. Eg. $2^3 + 1 = 10$ which is not prime	

Question Number	Please read notes for 8(i) before looking at scheme		Marks		
8.(i)	$8^{2x+1} = 6 \Rightarrow 2x+1 = \log_8 6$ M1 $\Rightarrow 2x+1 = \frac{\log_2 6}{\log_2 8}$ A1 $\Rightarrow 2x+1 = \frac{\log_2 2 + \log_2 3}{3}$ M1 $\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$ A1	$2^{6x+3} = 6$ $\Rightarrow (6x+3)\log_2 2 = \log_2 6$ M1 A1 $\Rightarrow (6x+3) = \log_2 2 + \log_2 3$ M1 $\Rightarrow x = \frac{\log_2 3}{6} - \frac{1}{3}$ A1	(4)		
(ii)	$\log_5 (7-2y) = 2\log_5 (y+1) - 1$ $\log_5 (7-2y) = \log_5 (y+1)^2 - 1$ $\log_5 (7-2y) = \log_5 (y+1)^2 - \log_5 5$ $(7-2y) = \frac{(y+1)^2}{5}$ $y^2 + 12y - 34 = 0 \Rightarrow y =$ $y = -6 + \sqrt{70}$ oe only	$2\log_5 (y+1) - \log_5 (7-2y) = 1$ $\log_5 (y+1)^2 - \log_5 (7-2y) = 1$ $\log_5 \frac{(y+1)^2}{(7-2y)} = 1$ $\frac{(y+1)^2}{(7-2y)} = 5$ $y^2 + 12y - 34 = 0 \Rightarrow y =$	M1 dM1 A1 ddM1 A1 (5) (9 marks)		
Notes					
<p>There are many different ways to attempt this but essentially can be marked in a similar way.</p> <p>If index work is used marks are not scored until the log work is seen</p> <p>Eg 1: $8^{2x+1} = 6 \Rightarrow 8^{2x} \times 8 = 6 \Rightarrow 8^{2x} = \frac{3}{4}$.</p> <p>1ST M1 is scored for $2x = \log_8 \frac{3}{4}$ and then 1ST A1 for $2x = \frac{\log_2 \frac{3}{4}}{\log_2 8}$</p> <p>but BOTH of these marks would be scored for $2x \log_2 8 = \log_2 \frac{3}{4}$</p> <p>2nd M1 would then be awarded for $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$</p> <p>Two more examples where the candidate initially uses index work.</p>					
<table border="1"> <tr> <td> $8^{2x+1} = 6 \Rightarrow 2^{3(2x+1)} = 6$ $3(2x+1) = \log_2 6$ is M1 A1 as it is a correct linear equation in x involving a log 2 term </td> <td> $8^{2x+1} = 6 \Rightarrow 64^x = \frac{3}{4}$ $\Rightarrow x = \log_{64} \frac{3}{4}$ is M1 But $\Rightarrow x \log_2 64 = \log_2 \frac{3}{4}$ is M1 A1 </td> </tr> </table>				$8^{2x+1} = 6 \Rightarrow 2^{3(2x+1)} = 6$ $3(2x+1) = \log_2 6$ is M1 A1 as it is a correct linear equation in x involving a log 2 term	$8^{2x+1} = 6 \Rightarrow 64^x = \frac{3}{4}$ $\Rightarrow x = \log_{64} \frac{3}{4}$ is M1 But $\Rightarrow x \log_2 64 = \log_2 \frac{3}{4}$ is M1 A1
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